Bio-mathematical Prey-Predator Model with Marine Protect Area(MPA) and Harvesting

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Abstract: This paper has formulated and discussed a prey-predator fishery model with marine protect area (MPA) along with combined and selective harvesting. Marine Protect area (MPA) consists of two zones namely unreserved zone (harvesting is not restricted) and reserved zone (harvesting is restricted). Biological equilibria of the system are derived, and the criteria for local stability, instability and global stability are established. The role of marine reserve is analyzed through the obtained results of numerical simulation through computer and MATLAB software of the proposed mathematical model. The results describe that size and existence of marine reserve have a stabilizing effect on population dynamics.

Keywords: Marine Protect Area(MPA), Combined harvesting, Selective harvesting, Reserved area, Unreserved area.

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1. Introduction:

Bio-mathematical model of unregulated and extensive exploitation of biological resources, such as fisheries and forestries, has obtained importance now-a-days. In last few decades, there is also a great interest on studying the co-existence of interacting biological species that has extensively been studied using mathematical model. Extensive and unregulated exploitation of several marine fishes in open access fishery has depleted the population of these fishes, some of which are in extinction.

To overcome this situation, several measures as restriction on harvesting, creating reserved zones, have been discussed with the technical issues related to the bio-economic exploitation of these resources. Several efficient management plains have been proposed for the improvement of renewable resources in such a way, that it would maintain the sustainable development and exploitation of these resources. One of the potential solutions is to create marine reserved zone where harvesting and other exploitation activities are restricted. Marine reserve zone not only preserves marine species inside the reserved zone, but it also enhances fish abundance in adjoining areas. Administer must consider the relationship between fishes and their neighboring habitat to effect the marine reserve zone.

Harvesting has a great effect on population dynamics of harvested species. Population dynamics with harvesting has been studied in mathematical Bio-economics. Harvesting and exploitation of biological resources are commonly used in fisheries, forestry and wild life management. Exploitation of multispecies systems are interesting and difficult both theoretically and practically.

Selective harvesting occurs when one of the multispecies is harvested and combined harvesting occurs when both of the multispecies are harvested. Several authors have studied bio-mathematical model with selective and combined harvesting. Clark[1] has investigated the problem of combined harvesting of independent species governed by the logistic law of growth. Brauer and Soudack[2, 3], Dai and Tang[4] and Myerscough et. al.[5] have studied predator-prey

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system with the effects of constant rate harvesting and stocking. Tuand Ho[6] have studied the stability of the unique positive equilibrium point of a nonselective harvesting of a predator-prey system with time delay. Kar and Matsuda [7] have considered a prey-predator model with Holling type of predation and harvesting of mature predator species.

The vital role of reserved zone in aquatic environment for protection of fishery resources from its overexploitation is discussed by several researchers. Arni et.al.[8] and Kar and Matsuda[9] have studied the use of marine protect area on both biological and economical aspect. Kar and Misra[10] have developed a prey-predator fishery model with influence of prey reserve and also noted that the fish population maintains at an equilibrium level in absence or presence of predator provided the population in the unreserved area lies in a certain interval. Srinivasu and Gayatri[11] have investigated the dynamics of prey and predator population which has been modeled for the situation in which a reserve is created to protect a certain number. Dubay et. al. [12] have proposed and analyzed a fishery resources system with reserved area. He has also investigated optimal harvesting policy of this system. Dubay[13] has developed a model to study the role of a reserved zone on the dynamics of prey-predator system and also established that the reserved zone has a stabilizing effect on predator-prey interactions. Conrad[14] has observed that a reserved zone generates no economic benefits to fisherman in absence ecological variability and optimal harvesting. This result agrees with the perspectives of many fishermen and also some economist. Luke et. al.[15] have showed that MPA's can be viewed as a kind of insurance against scientific uncertainty, stock assessments or regulation errors. The prey species which is to be conserved can be protected from predators by creating an artificial boundary of shelter that will divide the habitat into two zones – one reserved and other unreserved. The entry of predators into reserved zone can be restricted by the artificial boundary that may be in the form of fencing of suitable mesh size through which prey can pass but predator cannot. Kar and Chakraborty[16] have developed a model to examine the effects of marine reserves on equilibrium levels of fish biomass, catch, predation and rent in a fishery. They have also considered the optimal area of the reserve and exploitation rate in the fishery. Olivares and Arcos[17] have discussed a two patch model with marine protect area in bio-economic perspective. Louartassi et. al.[18] have studied the stability and static Output Feedback Design for a model the fishery resource with reserve area. Chakraborty and Kar[19] have studied a bio-economic model of a prey-predator fishery with protected area and also discussed the system numerically and observed that marine protect area can be used as an effective management tool to improve resource rent under a number of circumstances.

In this paper, a mathematical model is proposed and analyzed to study the dynamics of a prey-predator fishery resource system in an aquatic environment that consists of two zones: a free fishing zone and a reserved zone where fishing is strictly prohibited. Harvesting of prey and predator in unreserved area i.e. combined harvesting and harvesting of prey in unreserved area or harvesting of predator in unreserved area i.e. selective harvesting is considered. Biological equilibria of the system are derived and criteria for their stabilities are established. Numerical simulations are carried out through computer and MATLAB software to illustrate the feasibility of the dependence of the dynamic behavior with size and presence of reserve area along with combined and selective harvesting.

2. Model Formulation:

Consider a fishery habitat in an aquatic eco-system where prey and predator species are living together.

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It is assumed that the habitat is divided into two zones, namely, reserved and unreserved zones. It is assumed that predator species are not allowed to enter inside the reserved zone where as the free mixing of prey species from reserved to unreserved zone and vice-versa is permissible. Harvesting of prey and predator in unreserved area i.e. combined harvesting and harvesting of prey or predator in unreserved area i.e. selective harvesting are considered.

Let x(t) be the density of prey species in unreserved zone, y(t) be the density of prey species in reserved zone and z(t) be the density of the predator species at any time $t \geq 0$.. It is assumed that prey species in both zones are growing logistically and considered Holling type-II predator response function.

Now describe the model with two different cases

Case – I: Model with Combined harvesting

$$\frac{dx}{dt} = rx\left(1-\frac{x}{K}\right) - \sigma_1 x + \sigma_2 y - \frac{mxz}{a+x} - q_1 Ex$$

$$\frac{dy}{dt} = sy\left(1-\frac{y}{L}\right) + \sigma_1 x - \sigma_2 y$$

$$\frac{dz}{dt} = -dz + \frac{nxz}{a+x} - q_2 Ez ,$$

With $x(0) > 0$, $y(0) > 0$ and $z(0) > 0$ (2.1)

Case – II: Model with Selective harvesting

A: Harvesting of Prey only

$$\frac{dx}{dt} = rx\left(1-\frac{x}{K}\right) - \sigma_1 x + \sigma_2 y - \frac{mxz}{a+x} - q_1 Ex$$

$$\frac{dy}{dt} = sy\left(1-\frac{y}{L}\right) + \sigma_1 x - \sigma_2 y$$

$$\frac{dz}{dt} = -dz + \frac{nxz}{a+x} ,$$

With $x(0) > 0$, $y(0) > 0$ and $z(0) > 0$ (2-1a)

B: Harvesting of Predator only

$$\frac{dx}{dt} = rx\left(1-\frac{x}{K}\right) - \sigma_1 x + \sigma_2 y - \frac{mxz}{a+x}$$

$$\frac{dy}{dt} = sy\left(1-\frac{y}{L}\right) + \sigma_1 x - \sigma_2 y$$

$$\frac{dz}{dt} = -dz + \frac{nxz}{a+x} - q_2 Ez ,$$

With $x(0) > 0$, $y(0) > 0$ and $z(0) > 0$ (2.1b)

Where r and s are the intrinsic growth rates of prey sub population inside the unreserved and reserved area respectively, K and L are the carrying capacities of the prey species in the unreserved and reserved areas respectively, q_1 and q_2 are the catchability coefficient of prey and predator species in unreserved zone respectively, m and n are the conversion rate of prey by the predator and consume rate of predator of prey species respectively, a is the half saturation constant and E is the total effort applied for harvesting the prey and predator population in the unreserved zone. σ_1 is the rate at which the prey sub-population of unreserved area migrate into reserved area , σ_2 is the rate at which the prey subpopulation of reserved area migrate into unreserved area. The parameters r, s, σ_1, σ_2, K, L, q_1, q_2, a, m, n and E are assumed to be positive constants.

We note that if there is no migration of prey population from reserved zone to unreserved zone (i. e. $\sigma_2 = 0$) and

$r - \sigma_1 - q_1 E < 0$, then $\frac{dx}{dt} < 0$. (For case – I and case – IIA)

$r - \sigma_1 < 0$, then $\frac{dx}{dt} < 0$. (For case - IIB)

Similarly, if there is no migration of fish population from unreserved zone to reserved zone (i.e. $\sigma_1 = 0$) and $s - \sigma_2 < 0$, then $\frac{dy}{dt} < 0$. Hence throughout our analysis we assumed that

$r - \sigma_1 - q_1 E > 0$ (For case – I and case - IIA) (2-2a)

$r - \sigma_1 > 0$ (For case - IIB) (2-2b) and $s - \sigma_2 > 0$.(For all cases) (2.2c)

Now, the existence and local stability of equilibria, global stability and boundedness of the solution are discussed for the system (2.1). The systems (2.1a) and (2.1b) are not discussed separately as the analysis are similar to the system (2.1) by adjusting the parameter $q_1 = 0$ or $q_2 = 0$.

3. Existence of equilibria

Equilibria of model (2.1) are obtained by solving $\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$. It can be verified that model (2.1) has only three nonnegative equilibria, namely $P_0(0, 0, 0)$, $P_1(x_1, y_1, 0)$ and $P^*(x^*, y^*, z^*)$. The equilibrium P_0 exists obviously and the existence of P_1 and P^* are discussed as follows

Existence of $P_1(x_1, y_1, 0)$:

Here x_1 and y_1 are the positive solution of the following algebraic equations:

$$\sigma_2 y = -rx\left(1 - \frac{x}{K}\right) - \sigma_1 x - q_1 E x \quad (3.1a)$$

$$\sigma_1 x = sy\left(1 - \frac{y}{L}\right) - \sigma_2 y \quad (3.1b)$$

From equation (3.1a), we have

$$y = \frac{1}{\sigma_2}\left[\frac{rx^2}{K} - (r - \sigma_1 - q_1 E)x\right] \quad (3.2)$$

Substituting the value of y into (3.1b), we get a cubic equation in x as

$$Ax^3 + Bx^2 + Cx + D = 0. \quad (3.3)$$

Where

$$A = \frac{sr^2}{L\sigma_2^2 K^2}$$

$$B = \frac{-2sr(r - \sigma_1 - q_1 E)}{L\sigma_2^2 K^2}$$

$$C = \frac{s(r - \sigma_1 - q_1 E)^2}{L\sigma_2^2} - \frac{(s - \sigma_2)r}{K\sigma_2}$$

$$D = \frac{(s - \sigma 2)}{\sigma_2}(r - \sigma_1 - q_1 E) - \sigma_1 \quad (3.4)$$

By Descartes rule of sign, the above equation has a unique positive solution $x = x_1$ if the following inequalities hold: $\frac{s(r - \sigma_1 - q_1 E)^2}{L\sigma_2^2} < \frac{(s - \sigma 2)r}{K\sigma_2}$ (3.5a)

$$(s - \sigma_2)(r - \sigma_1 - q_1 E) < \sigma_1\sigma_2 \quad (3.5b)$$

Knowing the value of x_1 from (3.3), the value of y_1 can be computed from equation (3.2). It may also be noted that for y_1 to be positive, we must have

$$x_1 > \frac{K}{r}(r - \sigma_1 - q_1 E) \quad (3.6)$$

It is observed that existence of $P_1(x_1, y_1, 0)$ does not depend on harvesting of predator population but not on prey harvesting in unreserved area.

Existence of $P^*(x^*, y^*, z^*)$:

Here x^*, y^*, z^* are the positive solutions of the following algebraic equations:

$$rx\left(1 - \frac{x}{K}\right) - \sigma_1 x + \sigma_2 y - \frac{mxz}{a+x} - q_1 E x = 0 \quad (3.7a)$$

$$sy\left(1 - \frac{y}{L}\right) + \sigma_1 x - \sigma_2 y = 0 \quad (3.7b)$$

$$-dz - \frac{nxz}{a+x} - q_2 E z = 0 \quad (3.7c)$$

From (3.7c), $x^* = \frac{a(d + E q_2)}{n - d - Eq_2}$ (3.8)

From (3.7b) $\frac{sy^{*2}}{L}$ - (s - σ_2) y^* - $\sigma_1 x^*$ = 0

This equation has one change of sign as s > σ_2 (by (2.2)). So one positive root y^* must exist and the solution is

$$y^* = \frac{1}{2s}\left[(s - \sigma_2)L + \sqrt{(s - \sigma_2)^2L^2 + 4\,\sigma_2 x^* Ls}\right] \quad (3.9)$$

Again from (3.7a)

$$z^* = \frac{(a+x^*)}{mx^*}\left[\left(r - \sigma_1 - q_1E - \frac{r\,x^*}{K}\right)x^* + \sigma_2\, y^*\right] \quad (3.10)$$

Therefore $x^* > 0$ and $z^* > 0$ if $n > d + Eq_2$ and

$$r > \sigma_1 + q_1E + \frac{r\,x^*}{K} \quad (3.11)$$

Determining the value of x^* from (3.8), the value of y^* can be obtained from (3.9) and also the value of z^* can be obtained from (3.10).

It is observed that the existence of $P^*(x^*, y^*, z^*)$ depends on predator and prey harvesting and size of reserved area.

4. Boundedness of the solution:

In the following Lemma, it is established that all solutions of model (2.1) are nonnegative and bounded.

Lemma – 1 .

The set $\Omega = \left\{(x,y,z) \in \Re^3 : 0 < w = x + y + \frac{m}{n}z \le \frac{\mu}{\eta}\right\}$ is a region of attraction for all solutions initiating in the interior of the positive octant, where η is a positive constant and

$$\mu = \frac{K}{4r}(r + \eta - q_1E)^2 + \frac{L}{4S}(s + \eta)^2 + \frac{m}{2nd}[(\eta - d - q_1E)^2 + d^2z^2] \quad (4.1)$$

Proof:

Let $w = x + y + \frac{m}{n}z$ and $\eta > 0$ be a constant. Then

$$\frac{dw}{dt} + \eta\, w = x\left[r\left(1 - \frac{x}{K}\right) + \eta - q_1E\right] + y\left[r\left(1 - \frac{y}{K}\right) + \eta\right] + z\left[\frac{m}{n}(\eta - d - q_2E)\right] \quad (4.2)$$

From $[K(r + \eta - q_1E) - 2rx]^2 \ge 0$ it is $x\left[r\left(1 - \frac{x}{K}\right) + \eta - q_1E\right] \le \frac{K}{4r}(r + \eta - q_1E)^2$

And $[L(s + \eta) - 2sy]^2 \ge 0$ gives $y\left[r\left(1 - \frac{y}{K}\right) + \eta\right] \le \frac{L}{4S}(s + \eta)^2$

And $[(\eta - d - q_2E) - dz]^2 \ge 0$ gives $z\left[\frac{m}{n}(\eta - d - q_2E)\right] \le \frac{m}{2nd}[(\eta - d - q_1E)^2 + d^2z^2]$

Using these inequalities

$$\frac{dw}{dt} + \eta\, w \le \frac{K}{4r}(r + \eta - q_1E)^2 + \frac{L}{4S}(s + \eta)^2 + \frac{m}{2nd}[(\eta - d - q_1E)^2 + d^2z^2] = \mu(\text{say})$$

By using the differential inequality[20], obtain

$$0 < w(x(t), y(t), z(t)) \le \frac{\mu}{\eta}(1 - e^{-\eta t}) + (x(0), y(0), z(0))e^{-\eta t}$$

Taking limit $t \to \infty$, we have, $0 < w(t) \le \frac{\mu}{\eta}$,

Hence prove the lemma.

5. Local stability analysis:

Using the variational marix the local stability of the three equilibrium points are to be established

Stability of $P_0(0, 0, 0)$:

The corresponding variational matrix of the equilibrium point is

$$V_0(0,0,0) = \begin{pmatrix} -\sigma_1 & \sigma_2 & 0 \\ \sigma_1 & s-\sigma_2 & 0 \\ 0 & 0 & -d-q_2E \end{pmatrix} \quad (5.1)$$

The characteristic equation of the variational matrix is

$$\begin{vmatrix} -\sigma_1-\lambda & \sigma_2 & 0 \\ \sigma_1 & s-\sigma_2-\lambda & 0 \\ 0 & 0 & -d-q_2E-\lambda \end{vmatrix} = 0 \quad (5.2)$$

gives

$$(-\sigma_1-\lambda)(s-\sigma_2-\lambda)(-d-q_2E-\lambda) = 0$$

i.e. $\lambda = -\sigma_1, (s-\sigma_2), -(d+q_2E)$

therefore the eigen values of the characteristic equation are not all negative by (2.2c) , so the trivial solution $P_0(0, 0, 0)$ is unstable

Stability of $P_1(x_1, y_1, 0)$:

The corresponding variational matrix of the equilibrium point is

$$V_1(x_1, y_1, 0) = \begin{pmatrix} -A & \sigma_2 & -\frac{mx_1}{(a+x_1)} \\ \sigma_1 & -B & 0 \\ 0 & 0 & C \end{pmatrix} \quad (5.3)$$

Where $A = \frac{\sigma_2 y_1}{x_1} + \frac{rx_1}{K}$, $B = \frac{\sigma_1 x_1}{y_1} + \frac{sy_1}{L}$ and $C = \frac{nx_1}{(a+x_1)} - d - q_2E$

The characteristic equation of the variational matrix is

$$\begin{vmatrix} -A-\lambda & \sigma_2 & -\frac{mx_1}{(a+x_1)} \\ \sigma_1 & -B-\lambda & 0 \\ 0 & 0 & C-\lambda \end{vmatrix} = 0 \text{ gives} \quad (5.4)$$

$$(C-\lambda)\,[(A+\lambda)(B+\lambda)-\sigma_1\sigma_2] = 0$$

i.e. $\lambda = C$, will be negative if $\frac{nx_1}{(a+x_1)} < d + q_2E$ (5.5a)

and $\lambda^2 - (A+B)\lambda + AB - \sigma_1\sigma_2 = 0$

By Routh Howritz criteria the quadratic equation has negative roots

if $AB > \sigma_1\sigma_2$ (5.5b)

Therefore the roots of the characteristic equation are negative roots or complex root with negative real part if (5.5a) and (5.5b) are hold. Therefore the equilibrium point P_1 is stable and otherwise it is unstable. Stability of this equilibrium point depends on size of reserve area and predator harvesting but on prey harvesting.

Stability of $P^*(x^*, y^*, z^*)$:

$$V_2(x^*, y^*, z^*) = \begin{pmatrix} -A & \sigma_2 & -\frac{mx^*}{(a+x^*)} \\ \sigma_1 & -B & 0 \\ \frac{naz^*x^*}{(a+x^*)^2} & 0 & 0 \end{pmatrix} \quad (5.6)$$

The characteristic equation of the variational matrix is

$$\begin{vmatrix} -A_1-\lambda & \sigma_2 & -\frac{mx^*}{(a+x^*)} \\ \sigma_1 & -B_1-\lambda & 0 \\ \frac{naz^*x^*}{(a+x^*)^2} & 0 & -\lambda \end{vmatrix} = 0 \quad (5.7)$$

Where $A_1 = \frac{\sigma_2 x^*}{x^*} + \frac{rx^*}{K}$ and $B_1 = \frac{\sigma_1 x^*}{y^*} + \frac{sy^*}{L}$

$$\lambda^3 + (A_1+B_1)\lambda^2 + \left(\frac{A_1B_1 + mnaz^*x^*}{(a+x^*)^3} - \sigma_1\sigma_2\right)\lambda + \frac{mnay^*x^*}{(a+x^*)^3}B_1 = 0 \quad (5.8)$$

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By Routh Huwritz criteria the variational matrix has negative or complex with negative real part eigenvalues if

$$\frac{A_1{}^2B_1 + A_1 mnaz^* x^* + A_1 B_1{}^2}{(a+x^*)^3} > (A_1 + B_1)\sigma_1\sigma_2 \quad (5.9)$$

Therefore the equilibrium point P^* is asymptotically stable if (5.9) holds and otherwise it is unstable. Stability of this equilibrium point depends on the size of reserved area.

6. Global stability:

Theorem2: The interior equilibrium $P^*(x^*, y^*, z^*)$ is globally asymptotically stable with respect to all solutions initiating in the interior of the positive octant.

Proof:

Consider the following positive definite function about $P^*(x^*, y^*, z^*)$,

$$V(t) = \left(x - x^* - x^* \ln\frac{x}{x^*}\right) + \frac{y^*\sigma_2}{x^*\sigma_1}\left(y - y^* - y^* \ln\frac{y}{y^*}\right) + \frac{m}{n}\left(z - z^* - z^* \ln\frac{z}{z^*}\right) \quad (6.1)$$

Differentiating V(t) with respect to t along the solutions of (2.1) and simple algebraic manipulation we get

$$\frac{dV(t)}{dt} = -\frac{r}{K}(x - x^*)^2 - \frac{s}{L}(y - y^*)^2 - \frac{\sigma_2}{xyx^*}(yx^* - xy^*)^2 \quad (6.2)$$

Which is negative definite. Hence V(t) is a Liapunov function[21] with respect to $P^*(x^*, y^*, z^*)$whose domain contains the region of attraction Ω, proving the theorem.

7. Numerical results

In this section numerical simulation is present to illustrate the results obtained in previous sections. Now choose the following values of parameters

r = 0.4, s = 0.7, K = 400, σ_1 = 0.03, σ_2 = 0.02, m = 0.04, n = 0.06, E= 20, a = 50, d = 0.001, q_1 0.01, q_2 = 0.001.

First study the behavior of prey population in reserved and unreserved area and predator population in unreserved zone for various sizes of reserved area. Draw fig-1and fig-2 for L= 20 and fig-3 and fig-4 for L=150. Fig-1 represents the periodic orbit near the equilibrium point (P^*) and fig-2 depicts the oscillatory behavior of prey and predator populations in unreserved area throughout the interval except prey in reserved area. Therefore the system is unstable. A spiral phase portrait converging to the equilibrium point (P^*) in fig-3 denotes that the equilibrium point is asymptotically stable and fig-4 describes that both populations are attain their equilibrium level after some time interval and hence the system is stable

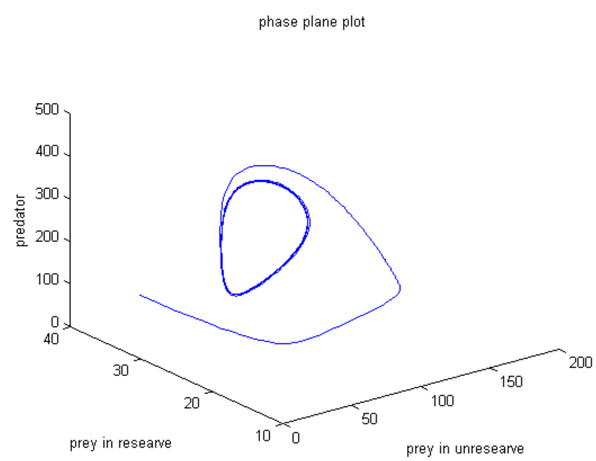


Fig -1: Phase portrait of the system showing a periodic orbit near $P^*(x^*, y^*, z^*)$

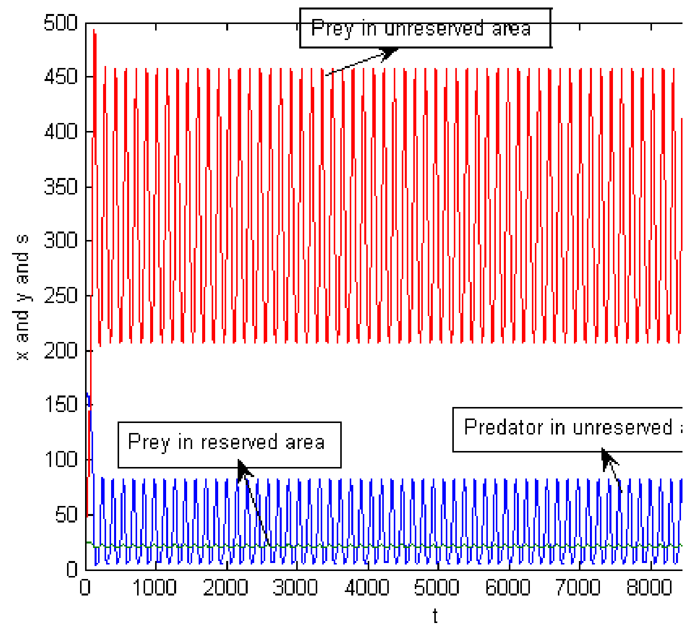


Fig-2: Oscillation of prey and predator in unreserved zone

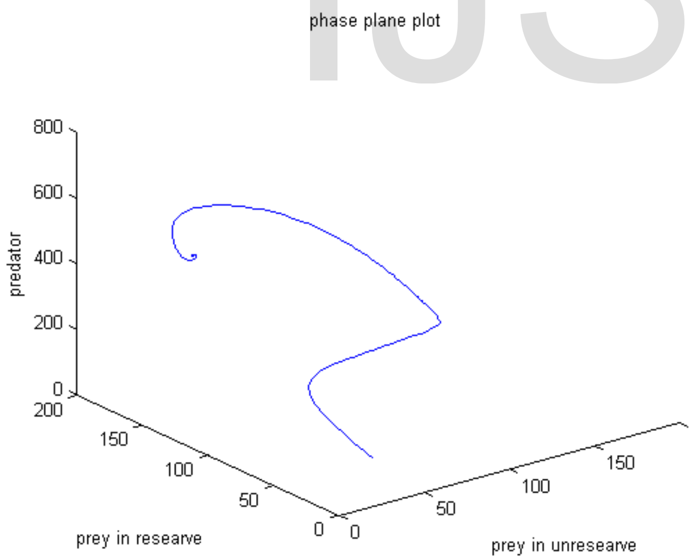


Fig – 3: Phase portrait of the system showing that $P^*(x^*, y^*, z^*)$ is locally asymptotically stable

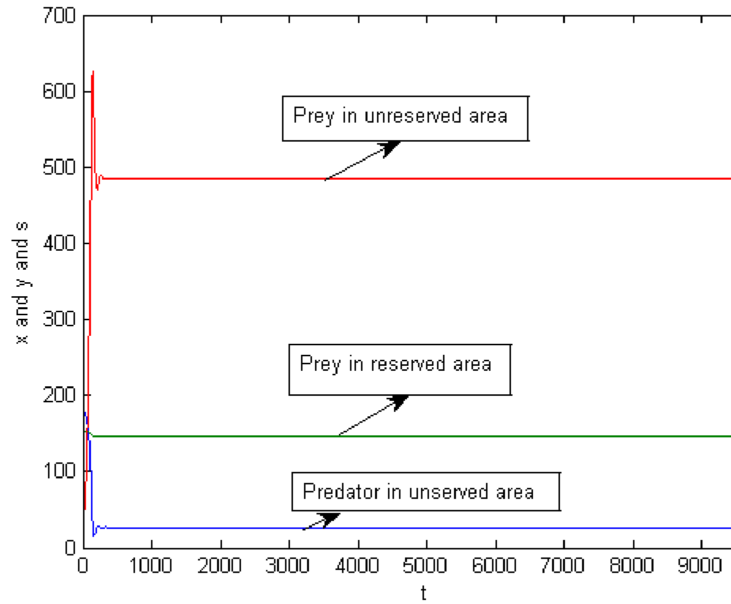


Fig – 4: Both populations converge to their equilibrium value

Now illustrate the behavior of Prey in reserved and unreserved zone and predator in unreserved zone with or without reserved area along with combined and selective harvesting with same parameter values. The behavior of these populations with or without reserved area in combined harvesting represents in fig-5 and fig-6. Fig-5 evokes that the system is stable as both populations converge to their equilibrium value after finite time oscillation with the appearance of reserved area and fig-6 illustrates that the system is unstable due to oscillation of these populations throughout the time interval with absence of reserved area.

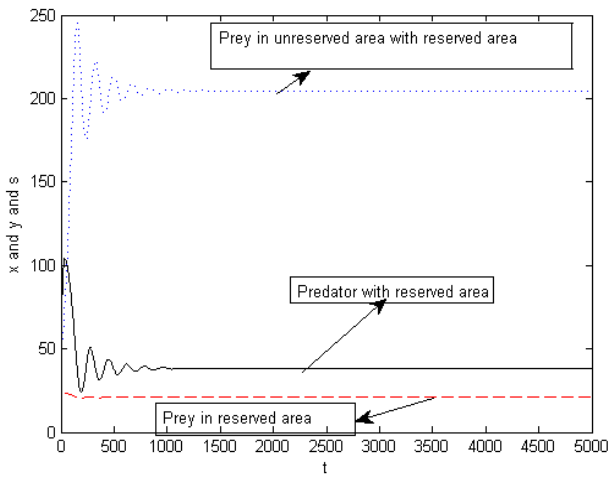


Fig – 5: Both populations converge to their equilibrium state value in combined harvesting with reserved area

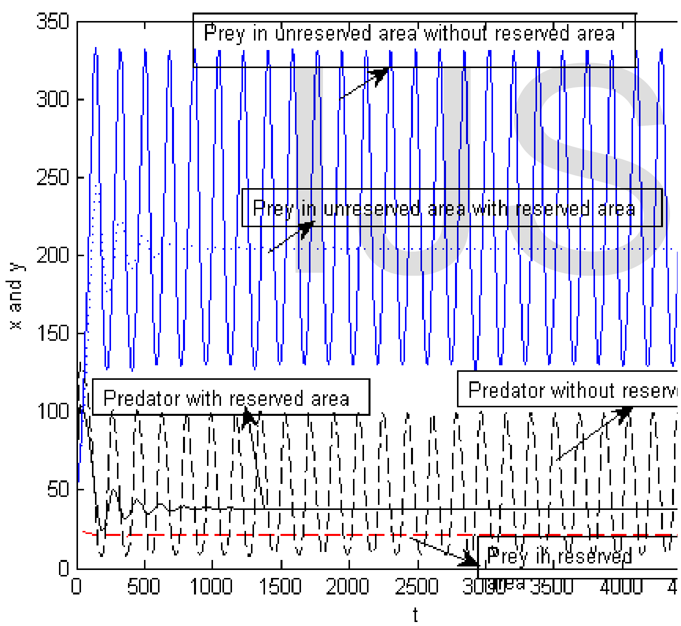


Fig-6: Oscillation of prey and predator in unreserved zone in combined harvesting without reversed area

Behavior of prey in reserved and unreserved zone and predator in unreserved zone in prey harvesting only(i.e. selective harvesting) with or without reserved area represents in fig-7 and fig-8. The prey-predator system is stable with the presence of reserved area as both populations achieve their steady state value after some finite interval from fig-7 and from fig-8, the oscillatory behavior of prey and predator in unreserved area without reserved area is observed and the system is unstable.

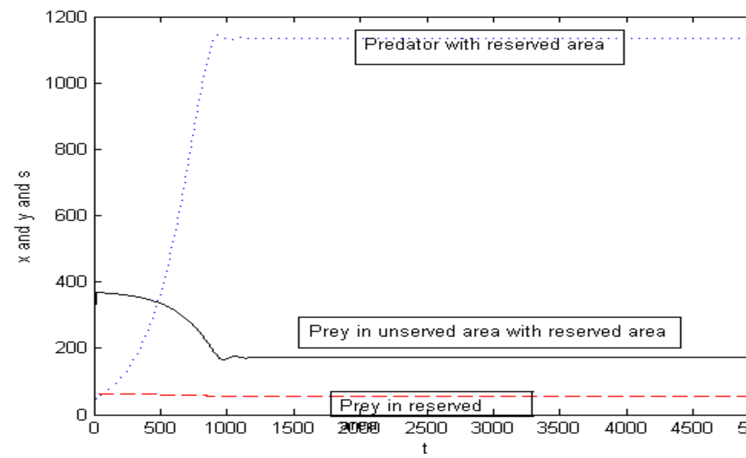


Fig – 7: Both populations converge to their equilibrium state with reserved area value along with selective harvesting(prey harvesting only)

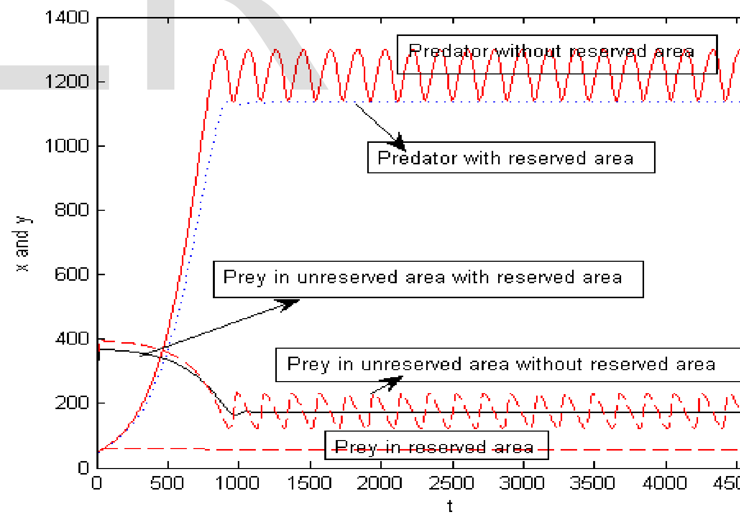
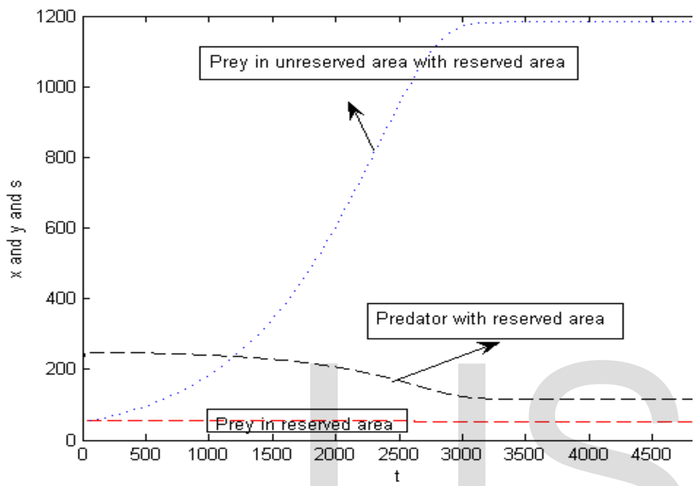


Fig-8: Oscillation of prey and predator in unreserved zone without reverse area along with selective harvesting(prey harvesting only)

Again the behavior of these populations with predator harvesting only (i.e. selective harvesting) along with or without reserved area is describe in fig-9 and fig-10. This has been done with the same parametric values as before except $q_2 = 0$. The prey-predator system is stable with the presence of reserved area as both populations attain their steady state value after some finite interval and it is observed from fig-9 and from fig-10, the oscillatory behavior of prey and predator in unreserved area with the absence of reserved area is observed and the system is unstable.



Predator harvesting with prey in reserve area

Fig – 9: Both populations converge to their equilibrium state value with selective harvesting(predator harvesting only) along

with reserved area

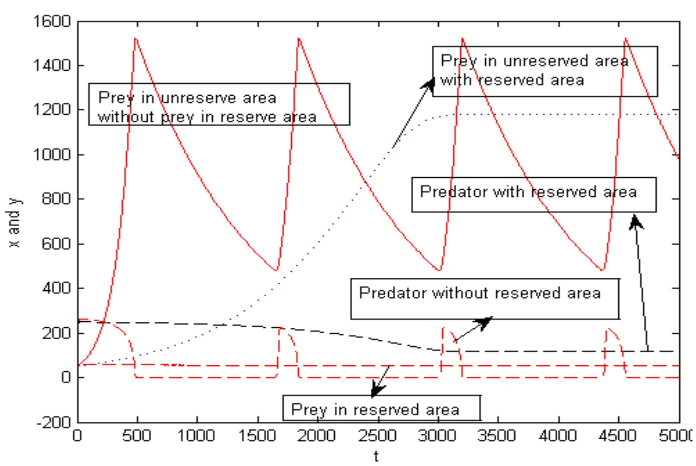


Fig – 10: Both populations converge to their equilibrium state value with selective harvesting(predator harvesting only) along without reserved area

8. Conclusion

In this paper, a mathematical prey-predator fishery model has been proposed and analyzed to study the role of a reserved zone with harvesting. Various outcomes due to the effect of reserved zone are discussed through numerical simulation with computer and MATLAB Software. These results are very much useful for resource conservation. It is observed that the unstable behavior of the system if the L(size of reserved area) = 20 and stable if L = 150. So the system transforms stable to unstable due to increasing of the size of reserved area and this analysis clearly expresses that size of the reserved zone has a stabilizing effect on the dynamics of prey-predator system. The obtained results also indicate that the system becomes unstable to stable both in combined and selective harvesting with the introduction of reserved zone. Therefore introduction of the reserved zone has also stabilized the fishery system.This study suggests that the role of reserved zone (either in size or existence) is an important integrating concept in ecology. By introducing reserved zone in fishery system where predator have no access, the prey species can grow without any external exploitations and hence prey species can be retained at an suitable level. This study is a management strategy to overcome this situation due to overexploitation problem ofmarine fishes in fishery management.

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